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(RESEARCH ARTICLE)

Orthogonal Jordan generalized triple higher reverse left (resp. right) centralizer on semiprime rings

Fawaz Ra'ad Jarullah *

Department of Financial and Banking Sciences, Imam Alkadhim University College, Iraq.

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Abstract

The indicate on this item to propose the entrance of orthogonal Jordan Generalized Triple Higher Reverse Left (resp.Right) Centralizers, in this paper (short by o. J. g. t)) on a ring E & survey this concept.

Keywords: 2-torsion free; Orthogonal Jordan; Triple Higher Reverse; Semiprime Rings

1. Introduction

The connotation of o. J. g . t is intrinsic in. In [1] &[2] show qualifier of semiprime (short by sp) & 2-torsion free (short by 2- tf) resp. on ring , adjacent for more preface see [3] , need the next :

Lemma (a): [2]

Let E is a 2-tf sp ring & $g, s \in E$, then the next demand are identical :

(i) $gws = 0$

(ii) $swg = 0$

(iii) $gws + swg = 0$, $\forall\ w \in E$

If one of these demand is accomplished ,then $gs = sg = 0$.

Lemma (b): [1]

Let E is a 2-tf sp ring & $g , s \in E$ if $gws + swg = 0$, $\forall\ w \in E$,then $gws = swg = 0$.

* Corresponding author: Dr. Fawaz Ra'ad Jarullah

2. Orthogonal Jordan Generalized Triple Higher Reverse Left (resp.Right) Centralizer on Semiprime Rings:

Qualifier (2.1):

Two J.g.t. $Y = (Y_i)_{i\in N}$ & $O = (O_i)_{i\in N}$ on a ring E are called **orthogonal** if $\forall$ g , s $\in$ E & n $\in$ N

$Y_n(g)\ E\ O_n(s) = (0) = O_n(s)\ E\ Y_n(g)$.

Lemma (2.2):

Let E is a 2-tf sp ring,$Y = (Y_i)_{i\in N}$ & $O = (O_i)_{i\in N}$ are o. J. g. t .Then $\dot{M}_n$ & $\dot{N}_n$ are orthogonal$\Leftrightarrow$ $Y_n(g)\ O_n(s) + O_n(g)\ Y_n(s) = 0$,

$\forall$ g , s $\in$ E & n $\in$ N .

Proof :

Attend Y_n & O_n are orthogonal

To manifest. $Y_n(g)\ O_n(s) + O_n(g)\ Y_n(s) = 0$

Seeing Y_n & O_n are orthogonal

$$\sum_{i=1}^{n} Y_i(g)\ w\ O_i(s) = 0 = \sum_{i=1}^{n} O_i(s)\ w\ Y_i(g)$$

By Lemma (a) , outcome

Another side , attend $Y_n(g)\ O_n(s) + O_n(g)\ Y_n(s) = 0$, $\forall$ g , s $\in$ E & n $\in$ N

To manifest Y_n & O_n are orthogonal

$$\sum_{i=1}^{n} Y_i(g)\ O_i(s) + O_i(g)\ Y_i(s) = 0$$

Left multiply by $y_{i-1}(w)$

$$\sum_{i=1}^{n} y_{i-1}(w)\ Y_i(g)\ O_i(s) + y_{i-1}(w)\ O_i(g)\ Y_i(s) = 0$$

Seeing Y_n & O_n are commuting

$$\sum_{i=1}^{n} Y_i(g)\ y_{i-1}(w)\ O_i(s) + O_i(g)\ y_{i-1}(w)\ Y_i(s) = 0$$

By Lemma (b) , outcome

Theorem (2.3):

Let E is a 2-tf sp ring ,$Y= (Y_i)_{i\in N}$ & $O = (O_i)_{i\in N}$ are o. J. g .t , when Y_n & O_n are commute. Then the therewith necessity are analogous, $\forall$ g , s $\in$ E & n$\in$N :

(i) $Y_n(g)\ O_n(s) = O_n(g)\ Y_n(s) = 0$

Hence $Y_n(g)\ O_n(s) + O_n(g)\ Y_n(s) = 0$

(ii) y_n , O_n are orthogonal & $y_n(g)\ O_n(s) = O_n(g)\ y_n(s) = 0$

(iii) o_n ,Y_n are orthogonal & $o_n(g)\ Y_n(s) = Y_n(g)\ o_n(s) = 0$

(iv) y_n ,o_n are orthogonal

Proof :

(i) Attend Y_n & O_n are orthogonal

$$\sum_{i=1}^{n} Y_i(g)\ w\ O_i(s) = 0 = \sum_{i=1}^{n} O_i(s)\ w\ Y_i(g)\ ,\ \forall\ g , s , w \in E$$

By Lemma (a) , outcome

(ii) Attend Y_n & O_n are orthogonal

By (i) $Y_n(g)\ O_n(s) = 0$

Left multiply by $y_{i-1}(w)$

$$\sum_{i=1}^{n} y_{i-1}(w)\ Y_i(g)\ O_i(s) = 0$$

By Lemma (a)

$$\sum_{i=1}^{n} y_{i-1}(w)\ O_i(s) = 0$$

Left multiply by $y_i(g)$

$$\sum_{i=1}^{n} y_i(g)\ y_{i-1}(w)\ O_i(s) = 0 \qquad \text{... (1)}$$

Seeing that O_n is a commute

$$\sum_{i=1}^{n} O_i(s)\ y_{i-1}(w)\ y_i(g) = 0 \qquad \text{... (2)}$$

By (1) & (2),outcome.

From (1)

$$\sum_{i=1}^{n} y_i(g)\, y_{i-1}(w)\, O_i(s) = 0$$

By Lemma (a) , outcome

(iii) Same way as (ii) .

(iv) Attend Y_n & O_n are orthogonal

By (ii)

$$\sum_{i=1}^{n} y_i(g)\, O_i(s) = 0$$

Move s by w

$$\sum_{i=1}^{n} y_i(g)\, O_i(w) = 0$$

Right multiply by $o_i(s)$

$$\sum_{i=1}^{n} y_i(g)\, O_i(w)\, o_i(s) = 0 \qquad \text{... (3)}$$

By (iii)

$$\sum_{i=1}^{n} o_i(g)\, Y_i(s) = 0$$

Move s by w

$$\sum_{i=1}^{n} o_i(g)\, Y_i(w) = 0$$

Right multiply by $y_i(s)$

$$\sum_{i=1}^{n} o_i(g)\, Y_i(w)\, y_i(s) = 0 \qquad \text{... (4)}$$

By (3) and (4) , outcome

Theorem (2.4):

Let E is a 2-tf sp ring , $Y = (Y_i)_{i\in N}$ & $O = (O_i)_{i\in N}$ are o. J. g. t .Then the therewith necessity are analogous, $\forall\, n\in N$:

(i) $y_nO_n = O_ny_n = 0$

(ii) $o_nY_n = Y_no_n = 0$

(iii) $Y_nO_n = O_nY_n = 0$

Proof :

(i) Attend Y_n & O_n are orthogonal

By Theorem (2.3)(ii)

$$\sum_{i=1}^{n} O_i(g)\, y_i(s) = 0$$

Right multiply by $O_i(g)$

$$\sum_{i=1}^{n} y_i(O_i(g)\, y_i(s)\, O_i(g)) = 0$$

$$\sum_{i=1}^{n} y_i(O_i(g))\, y_{i-1}(y_i(s))\, y_{i-1}(O_i(g)) = 0$$

Move $y_{i-1}(O_i(g))$ by $y_i(O_i(g))$

$$\sum_{i=1}^{n} y_i(O_i(g))\, y_{i-1}(y_i(s))\, y_i(O_i(g)) = 0$$

E is a sp ring

$$\sum_{i=1}^{n} y_i(O_i(g)) = 0 \Rightarrow y_n O_n = 0 \qquad \text{... (5)}$$

By Theorem (2.3)(ii)

$$\sum_{i=1}^{n} y_i(g)\, O_i(s) = 0$$

Right multiply by $y_i(g)$

$$\sum_{i=1}^{n} O_i(y_i(g)\, O_i(s)\, y_i(g)) = 0$$

$$\sum_{i=1}^{n} O_i(y_i(g))\, o_{i-1}(O_i(s))\, o_{i-1}(y_i(g)) = 0$$

Move $o_{i-1}(y_i(g))$ by $O_i(y_i(g))$

$$\sum_{i=1}^{n} O_i(y_i(g))\, o_{i-1}(O_i(s))\, O_i(y_i(g)) = 0$$

E is a sp ring

$$\sum_{i=1}^{n} O_i(y_i(g)) = 0 \quad \Rightarrow O_n\, y_n = 0 \qquad \dots (6)$$

From (5) & (6),outcome

(ii) Same way as (i)

(iii) Attend Y_n & O_n are orthogonal

By Theorem (2.3)(i)

$$\sum_{i=1}^{n} O_i(g)\, Y_i(s) = 0$$

Right multiply by $O_i(g)$

$$\sum_{i=1}^{n} Y_i(O_i(g)\, Y_i(s)\, O_i(g)) = 0$$

$$\sum_{i=1}^{n} Y_i(O_i(g))\, y_{i-1}(Y_i(s))\, y_{i-1}(O_i(g)) = 0$$

By Lemma (a)

$$\sum_{i=1}^{n} Y_i(O_i(g))\, y_{i-1}(O_i(g)) = 0$$

Right multiply by $Y_i(O_i(g))$

$$\sum_{i=1}^{n} Y_i(O_i(g))\, y_{i-1}(O_i(g))\, Y_i(O_i(g)) = 0$$

E is a sp ring

$$\sum_{i=1}^{n} Y_i(O_i(g)) = 0 \quad \Rightarrow Y_n\, O_n = 0 \qquad \dots (7)$$

By Theorem (2.3)(i)

$$\sum_{i=1}^{n} Y_i(g)\, O_i(s) = 0$$

Right multiply by $Y_i(g)$

$$\sum_{i=1}^{n} O_i(Y_i(g)\ O_i(s)\ Y_i(g)) = 0$$

$$\sum_{i=1}^{n} O_i(Y_i(g))\ o_{i-1}(O_i(s))\ o_{i-1}(Y_i(g)) = 0$$

By Lemma (a)

$$\sum_{i=1}^{n} O_i(Y_i(g))\ o_{i-1}(Y_i(g)) = 0$$

Right multiply by $O_i(Y_i(g))$

$$\sum_{i=1}^{n} O_i(Y_i(g))\ o_{i-1}(Y_i(g))\ O_i(Y_i(g)) = 0$$

E is a sp ring

$$\sum_{i=1}^{n} O_i(Y_i(g)) = 0 \Rightarrow O_n\ Y_n = 0 \qquad \dots (8)$$

From (7) & (8) , outcome

Theorem (2.5):

Let E is a 2-tf sp ring , $Y = (Y_i)_{i\in N}$ & $O = (O_i)_{i\in N}$ are J.g.t, when Y_n & O_n are commute. Then Y_n & O_n are orthogonal $\Leftrightarrow$ therewith necessity are analogous , $\forall$ g, q $\in$ E & n$\in$ N :

(i) $Y_n(g)\ O_n(s) + O_n(g)\ Y_n(s) = 0$

(ii) $y_n(g)\ O_n(s) + o_n(g)\ Y_n(s) = 0$

Proof :

Attend Y_n & O_n are orthogonal

To manifest **(i)** $Y_n(g)\ O_n(s) + O_n(g)\ Y_n(s) = 0$

(ii) $y_n(g)\ O_n(s) + o_n(g)\ Y_n(s) = 0$

Y_n & O_n are orthogonal

By Lemma (2.4) , outcome (i) .

By Theorem (2.3)(i)

$O_n(g)\ Y_n(s) = 0$

Left multiply by g

$$\sum_{i=1}^{n} y_i(g\, O_i(g)\, Y_i(s)\,) = 0$$

$$\sum_{i=1}^{n} y_i(Yi(s))\, y_{i-1}(O_i(g))\, y_{i-1}(g) = 0$$

Moving $y_{i-1}(O_i(g))$ by $O_i(s)$

$$\sum_{i=1}^{n} y_i(g)\, O_i(s)\, y_{i-1}(Y_i(s)) = 0$$

Right multiply by $y_i(g)\, O_i(s)$

$$\sum_{i=1}^{n} y_i(g)\, O_i(s)\, y_{i-1}(Y_i(s))\, y_i(g)\, O_i(s) = 0$$

E is a sp ring

$$\sum_{i=1}^{n} y_i(g)\, O_i(s) = 0 \Rightarrow y_n(g)\, O_n(s) = 0 \qquad \dots (9)$$

By Theorem (2.3)(i)

$Y_n(g)\, O_n(s) = 0$

Left multiply by g

$$\sum_{i=1}^{n} o_i(\, gY_i(g)\, O_i(s)) = 0$$

$$\sum_{i=1}^{n} o_i(j)\, o_{i-1}(Y_i(g))\, o_{i-1}(O_i(s)) = 0$$

Moving $o_{i-1}(Y_i(g))$ by $Y_i(s)$

$$\sum_{i=1}^{n} o_i(g)\, Y_i(s)\, o_{i-1}(O_i(s)) = 0$$

Right multiply by $o_i(g)\, Y_i(s)$

$$\sum_{i=1}^{n} o_i(g)Y_i(s)\, o_{i-1}(O_i(s))\, o_i(g)\, Y_i(s) = 0$$

E is a sp ring

$$\sum_{i=1}^{n} o_i(g)Y_i(s) = 0 \Rightarrow o_n(g)Y_n(s) = 0 \qquad \dots (10)$$

From (9) & (10) , outcome (ii) .

Another side , Attend requirement are hold

(i) $Y_n(g)\ O_n(s) + O_n(g)\ Y_n(s) = 0$

(ii) $y_n(g)\ O_n(s) + o_n(g)\ Y_n(s) = 0$

To manifest Y_n & O_n are orthogonal

From (i) & By Lemma (2.2), outcome

3. Conclusion

In this paper we defined the concept of Orthogonal Jordan Generalized Triple Higher Reverse Left (resp.Right) Centralizer on Rings on on Rings.We can find more results, properties, Lemmas and Theorems. According to the previous results, the future study in this paper will define on Gamma rings and Models.

Compliance with ethical standards

Disclosure of conflict of interest

There was no conflict of interest to be disclosed.

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